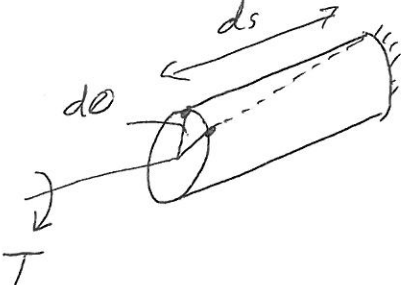




EXAMINATION SOLUTIONS: SESSION 2016/2017

School:	SCHOOL OF ENGINEERING		
Programme:	MECHANICAL & MANUFACTURING	Year:	3
Module/Unit:	ADVANCED STRESS ANALYSIS		
Author:	KELVIN LAMÉ	Page:	1 of 17

Question No	Question	Marks
1a _i	 SINCE $\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}$ $\theta = \frac{TL}{GJ}$ ∴ FROM DIAGRAM : $d\theta = \frac{Tds}{GJ}$	(2)

Continuation Sheet

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Question No	Question	Marks
(1a ii)	now $U = \frac{1}{2} T d\theta$ $\therefore$ FOR SECTION SHOWN $U = \frac{1}{2} T \cdot \frac{T ds}{GJ}$ $= \frac{T^2 ds}{GJ}$ $\therefore$ TOTAL STRAIN ENERGY $U = \int_0^L \frac{T^2 ds}{2GJ}$ $= \frac{T^2 L}{2GJ}$	(4)
(iii)	RESILIENCE = STRAIN PER UNIT VOLUME IF $U = \frac{P^2 L}{2AE}$ $U = \frac{\sigma^2 A^2 L}{2AE}$ $\therefore U = \frac{\sigma^2 AL}{2E}$ $\therefore U/AL = \frac{\sigma^2}{2E}$ OR $U/V = \frac{\sigma^2}{2E}$	(4)

Continuation Sheet

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Question No	Question	Marks
(bi)	$\sigma = \frac{F}{A} \quad \therefore F = \sigma A = 300 \times 10^6 \times (20 \times 20) \times 10^{-6}$ $= \underline{\underline{120 \text{ kN}}}$ $\text{AREA} = 20 \times 20 \times 10^{-6}$ $= \underline{\underline{4 \times 10^{-4} \text{ m}^2}}$ $\therefore U = \frac{P^2 L}{2AE} = \frac{120,000^2 \times 4}{2 \times 4 \times 10^{-4} \times 200 \times 10^9} = \underline{\underline{360 \text{ J}}}$	(3)
(ii)	$\text{P.E.} = mgh$ $= mg(h + \delta L)$ $E = \frac{\sigma}{\epsilon} = \sigma \div \frac{\delta L}{L} = \frac{\sigma L}{\delta L} \quad \therefore \delta L = \frac{\sigma L}{E}$ P.E. IS CONVERTED INTO STRAIN ENERGY $\therefore mg(h + \delta L) = \frac{P^2 L}{2AE} \quad \therefore \frac{\sigma^2 V}{2Emg} = h + \frac{\sigma L}{E}$ $mg\left(h + \frac{\sigma L}{E}\right) = \frac{\sigma^2 A L}{2AE} \quad \therefore h = \frac{\sigma^2 V}{2Emg} - \frac{\sigma L}{E}$ $\therefore mg\left(h + \frac{\sigma L}{E}\right) = \frac{\sigma^2 AL}{2E}$ $mg\left(h + \frac{\sigma L}{E}\right) = \frac{\sigma^2 V}{2E}$	(4)

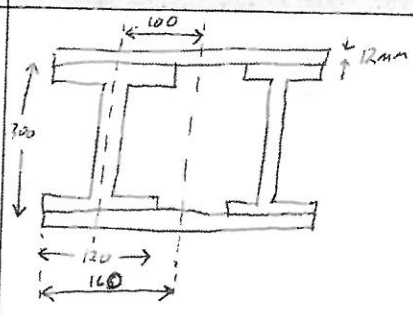
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Question No	Question	Marks
(biii)	$h = \frac{\sigma^2 L}{2E\pi g} - \frac{\sigma L}{E}$ $= \frac{(75 \times 10^6)^2 \times (4 \times 10^{-4} \times 4)}{2 \times 200 \times 10^9 \times 200 \times 9.81} - \frac{75 \times 10^6 \times 4}{200 \times 10^9}$ $= \frac{9 \times 10^{12}}{7.85 \times 10^{14}} - \frac{300 \times 10^6}{200 \times 10^9}$ $= 11.47 \times 10^{-3} - 1.5 \times 10^{-3}$ $= \underline{\underline{9.97 \text{ mm}}}$	

Continuation Sheet

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Question No	Question	Marks
2	 FOR EACH JOIST $I_{xx} = 96 \times 10^{-6} \text{ m}^4$ $I_{yy} = 4.8 \times 10^{-6} \text{ m}^4$ $A = 6 \times 10^{-3} \text{ m}^2$ XX: JOISTS $I_{xx} = 2 \times 96 \times 10^{-6} = \underline{192 \times 10^{-6} \text{ m}^4}$ PLATES: $I_{xx} = \square - \square$ $= \left[\frac{320 \times 324^3}{12} - \frac{320 \times 300^3}{12} \right] \times 10^{-12}$ $= \underline{187 \times 10^{-6} \text{ m}^4}$ $I_{\text{TOTAL}} = 187 \times 10^{-6} + 192 \times 10^{-6}$ $= \underline{379 \times 10^{-6} \text{ m}^4}$	

Continuation Sheet

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Question No	Question	Marks
2 (a) CONT	YY NOW $I_G = I_{NA} + Ah^2$ JOISTS: $I_{YY} = 2 (4.2 \times 10^{-6} + 6 \times 10^{-3} \times 0.1^2)$ $= 128.4 \times 10^{-6} \text{ m}^4$ PLATES $I_{YY} = \frac{BD^3}{12}$ $= 2 \left(\frac{12 \times 320^3}{12} \right) \times 10^{-12}$ $= 6.55 \times 10^{-5} \text{ m}^4$ $\therefore I_{YY \text{ total}} = 128.4 \times 10^{-6} + 6.55 \times 10^{-5}$ $= 193.9 \times 10^{-6} \text{ m}^4$ SO, ~~IT WILL BUCKLE ABOUT YY~~ $I_{xx} = 379 \times 10^{-6} > I_{YY} = 193.9 \times 10^{-6}$ $\therefore$ IT WILL BUCKLE ABOUT YY	(5)

Continuation Sheet

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Question No	Question	Marks
(b)	$K = \sqrt{\frac{I}{A}}$ $\text{TOTAL AREA} = 2 \times \text{TUBES} + 2 \times \text{PLATES}$ $= 2 \times 6 \times 10^{-3} + 2 (320 \times 12) \times 10^{-6}$ $= \underline{\underline{0.01968 \text{ m}^2}}$ $K = \sqrt{\frac{I}{A}} = \sqrt{\frac{193.9 \times 10^{-6}}{0.01968}}$ $= \underline{\underline{0.1 \text{ m}^2}}$	(2)
(c)	$\text{SLENDERNESS RATIO} = \frac{L}{K}$ $= \frac{7}{0.1}$ $= \underline{\underline{70}}$	(2)

Continuation Sheet

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Question No	Question	Marks
d)	$P_e = \frac{\pi^2 EI}{L^2}$ $= \frac{\pi^2 \times 200 \times 10^9 \times 193.9 \times 10^{-6}}{7^2}$ $= \cancel{7.8 \times 10^6} = \underline{\underline{7.8 \times 10^6 \text{ N}}}$	
e)	$P_R = \frac{\sigma_y A}{1 + a \left(\frac{L}{r} \right)^2}$ $= \frac{320 \times 10^6 \times 0.01968}{1 + \frac{1}{7500} \left(\frac{7}{0.1} \right)^2}$ $= \underline{\underline{3.8 \times 10^6 \text{ N}}}$	3 3

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Question No	Question	Marks
(2e)	$P_R = \frac{\sigma_y A}{1 + \lambda \left(\frac{L}{r}\right)^2}$ $= \frac{320 \times 10^6 \times 0.0148}{1 + \frac{1}{7500} \times 97.9^2}$ $= \underline{\underline{2.08 \times 10^6 \text{ N}}}$	(3)
(f)	FOR THIS STRUT, WHICH IS APPROACHING THE VALIDITY LIMIT FOR EULERS THEORY, THERE IS AN APPRECIABLE DIFFERENCE BETWEEN THE RESULTS GIVEN BY THE TWO THEORIES 	



3a

A FATIGUE FAILURE IS A FAILURE SEEN UNDER FLUCTUATING STRESS WHICH IS BELOW THE TENSILE STRENGTH OR EVEN THE ELASTIC LIMIT OF THE MATERIAL.

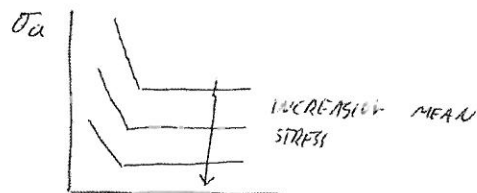
[2 MARKS]

(b) THE SURFACE OF A FATIGUE-FAILED COMPONENT TYPICALLY HAS 3 AREAS:

- (1) THE POINT OF INITIATION
- (2) "ARREST LINES" WHEN THE CRACK PROPAGATES
- (3) THE CRYSTALLINE AREA OF RUPTURE

[3 MARKS]

(c)

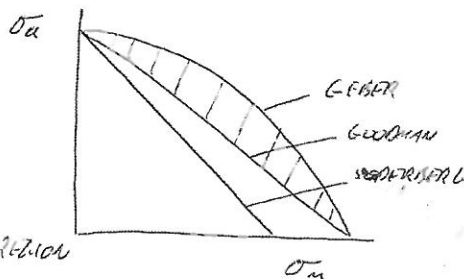


AS THE MEAN STRESS OF A SPECIMEN IS INCREASED, THEN THE FATIGUE LIFE IS REDUCED.

(d) THREE THEORIES ARE

- GOODMAN
- GEBER
- SODERBERG

[3 MARKS]



FAILURES ARE TYPICALLY SEEN IN THE SHADDED REGION BETWEEN GERBER & GOODMAN.

AS SUCH PEOPLE OFTEN USE SODERBERG AS IT IS THE MOST CONSERVATIVE.

[8 MARKS]



(e)

<u>CYCLES SEEN</u>	<u>CYCLES TO FAILURE</u>
3000 @ 510 MPa	: 10,050 at $\sigma_m = 0$
12,000 @ 450 MPa	: 50,500 " "
80,000 @ 380 MPa	: 275,000 " "
x @ 480 MPa	: 20,800 " "

(2)

FROM MINERS LAW

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} + \frac{n_4}{N_4} = 1$$

$$\frac{3000}{10,050} + \frac{12,000}{50,500} + \frac{80,000}{275,000} + \frac{x}{20,800} = 1$$

$$\therefore \frac{x}{20,800} = 1 - \left(\frac{3000}{10,050} + \frac{12,000}{50,500} + \frac{80,000}{275,000} \right)$$

$$= 1 - 0.8270$$

$$x = \underline{\underline{3598}} \text{ CYCLES}$$

[4 MARKS]

Continuation Sheet

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Question No	Question	Marks
(4) (2)	$\frac{\sigma}{y} = \frac{M}{I}$ $\therefore \sigma = \frac{My}{I}$ now $I_o = \frac{\pi d^4}{64}$ $\sigma = \frac{64My}{\pi d^4}$ $= \frac{64 \times 1.9 \times 10^3 \times 25 \times 10^{-3}}{\pi \times (50 \times 10^{-3})^4}$ $= \underline{\underline{155 \text{ MPa}}}$	(3)

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Question No	Question	Marks
6	$\frac{I}{J} = \frac{\tau}{R}$ $\therefore \tau = \frac{TR}{J}$ $\text{NOW } J_0 = \frac{\pi d^4}{32}$ $\therefore \tau = \frac{32TR}{\pi d^4}$ $= \frac{32 \times T \times 25 \times 10^{-3}}{\pi \times (50 \times 10^{-3})^4}$ $= \underline{\underline{40.7 \times 10^3 T}}$	3

Continuation Sheet

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Question No	Question	Marks
(C) (1)	MAX SHEAR STRESS THEORY $\sigma_1 - \sigma_3 = \sigma_y$ FROM COMPLEX STRESS $\sigma_1 = \frac{1}{2} [(\sigma_x + \sigma_y) + \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}]$ let $a = (\sigma_x + \sigma_y)$ $b = \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$ $\therefore \sigma_1 = \frac{1}{2}(a + b)$ $= \underline{\underline{\frac{1}{2}a + \frac{1}{2}b}}$ $\sigma_3 = \frac{1}{2} [(\sigma_x + \sigma_y) - \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}]$ $= \frac{1}{2}(a - b)$ $= \frac{1}{2}a - \frac{1}{2}b$ NOW $\sigma_1 - \sigma_3 = \frac{1}{2}a + \frac{1}{2}b - (\frac{1}{2}a - \frac{1}{2}b)$ $= b$ $= \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$	

Continuation Sheet

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Question No	Question	Marks
(C) (i) CONT	$\therefore \sigma_{yield} = \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$ SINCE $\sigma_y = 0$ $\sigma_{yield} = \sqrt{\sigma_x^2 + 4\tau_{xy}^2}$ $\therefore \tau_{xy} = \sqrt{\frac{\sigma_{yield}^2 - \sigma_x^2}{4}}$ $= \sqrt{\frac{(200 \times 10^6)^2 - (155 \times 10^6)^2}{4}}$ $= 63.2 \times 10^6 \text{ Pa}$ SINCE $\tau = 40.7 \times 10^3 \text{ T}$ $63.2 \times 10^6 = 40.7 \times 10^3 \text{ T}$ $\therefore T = 1.55 \text{ kNm}$	(7)

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Question No	Question	Marks
(C) ii	$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_y^2$ $\sigma_1 = \frac{1}{2}a + \frac{1}{2}b$ $\sigma_2 = 0$ $\sigma_3 = \frac{1}{2}a - \frac{1}{2}b$ $\left(\frac{1}{2}a + \frac{1}{2}b\right)^2 + \left(-\frac{1}{2}a + \frac{1}{2}b\right)^2 + \left(\frac{1}{2}a - \frac{1}{2}b - \frac{1}{2}a + \frac{1}{2}b\right)^2 = 2\sigma_y^2$ $\left(\frac{1}{2}a + \frac{1}{2}b\right)\left(\frac{1}{2}a + \frac{1}{2}b\right) + \left(-\frac{1}{2}a + \frac{1}{2}b\right)\left(-\frac{1}{2}a + \frac{1}{2}b\right) + b^2 = 2\sigma_y^2$ $\frac{1}{4}a^2 + \frac{1}{4}ab + \frac{1}{4}ab + \frac{1}{4}b^2 + \frac{1}{4}a^2 - \frac{1}{4}ab - \frac{1}{4}ab + \frac{1}{4}b^2 + b^2 = 2\sigma_y^2$ $\frac{1}{2}a^2 + \frac{3}{2}b^2 = 2\sigma_y^2$ now $a = \sigma_x + \sigma_y$ $= \sigma_x$ (since $\sigma_y = 0$) $b = \sqrt{\sigma_x^2 + 4\tau_{xy}^2}$	

Continuation Sheet

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Question No	Question	Marks
(C) (ii) CONT.	now $\frac{1}{2}a^2 + \frac{3}{2}b^2 = 2\sigma_y^2$ $\frac{1}{2}\sigma_x^2 + \frac{3}{2}(\sigma_x^2 + 4\tau_{xy}^2) = 2\sigma_y^2$ $\frac{1}{2}\sigma_x^2 + \frac{3}{2}\sigma_x^2 + 6\tau_{xy}^2 = 2\sigma_y^2$ $2\sigma_x^2 + 6\tau_{xy}^2 = 2\sigma_y^2$ $\sigma_x^2 + 3\tau_{xy}^2 = \sigma_y^2$ $\sigma_x = 155 \text{ MPa}$ $\tau_{xy} = 40.7 \times 10^3 \text{ Pa}$ $= 40.7 \times 10^{-3} \text{ T MPa}$ $\therefore 155^2 + 3(40.7 \times 10^{-3} T)^2 = 200^2$ $155^2 + 3(40.7 \times 10^{-3})^2 T^2 = 200^2$ $(40.7 \times 10^{-3})^2 T^2 = \frac{200^2 - 155^2}{3}$ $\therefore T = \frac{5325}{(40.7 \times 10^{-3})^2} \therefore T = 1.79 \text{ MN/m}^2$	